

THE EFFECT OF EDGE CONDITIONS ON NATURAL CONVECTION FROM A HORIZONTAL PLATE

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Abstract—Natural convection from a horizontal heated square plate facing downward in air was studied with three different edge conditions: vertical surfaces extending up from the plate edge cooled to ambient temperature, vertical surfaces extending up from the edge kept at the plate temperature and adiabatic horizontal extensions next to the outer edge of the plate. The local heat flux was found by measuring the temperature gradient at the plate surface. The average heat transfer coefficient was found from the local values.

The average heat transfer coefficient for the heated edge was seventeen percent greater than the heat transfer coefficient for the cooled edge. The heat transfer coefficient for the horizontal extensions was thirty percent below that for the cooled edge.

NOMENCLATURE

- g , acceleration of gravity [ft/s²];
 h , heat transfer coefficient [Btu/hft² °F];
 k , thermal conductivity [Btu/hft °F];
 L , length of test plate [ft];
 Nu_x , local Nusselt number, $hL/2k$;
 \overline{Nu}_x , Nusselt number averaged over plate area;
 Ra , Rayleigh number $g\beta \frac{(T_w - T_\infty)L^3}{\nu\alpha}$;
 T_w , wall temperature [°F];
 T_∞ , ambient temperature [°F];
 x , distance from centre of plate [ft];
 α , thermal diffusivity [ft²/h];
 β , coefficient of thermal expansion [1/°R];
 δ , thermal boundary layer thickness [ft];
 δ_1 , thermal boundary layer thickness at $x = 0$ [ft];
 ν , kinematic viscosity [ft²/h].

1. INTRODUCTION

THE SOLUTION for natural convection from vertical plates is well known. However, the problem of natural convection from a heated horizontal plate facing downward (or a cooled plate facing upward) has been the subject of considerable interest and controversy.

Wagner [1] applied the integral method of Levy [2] for two dimensional laminar flow to obtain one of the first solutions to the problem of horizontal plate. Wagner assumed the fluid flowed from the center of the plate to the edges and he also assumed that the boundary layer thickness at the edge can be set equal to zero. Stewartson [3] assumed the edges were stagnation points similar to inclined plates and the

flow was from the edge toward the center. Stewartson sites experimental evidence which shows a thicker boundary layer at the center than at the edge to justify his assumption. However, by ignoring vertical velocities it can be shown from hydrostatics that the pressure of the gas immediately under the heated plate is higher at the center where the thermal boundary layer is thicker than at the edge. This produces a horizontal pressure gradient which moves the flow from the center toward the edge. Gill, Zeh and del Casals [4] and Singh, Birkebak and Drake [5] have presented solutions which flow from the center toward the edge using the edge condition that the boundary layer has a zero thickness. Singh's work is for rectangular, square and round plates.

Singh and Birkebak [6] gave a two dimensional solution which includes non-zero boundary layer thickness at the edges. However, no account of pressure or velocity variation due to the change in the flow direction near the edge is considered. As the flow approaches the edge it is constrained to be horizontal by the plate surface, after passing the edge the flow turns to form a vertical plume. Clifton and Chapman [7] performed an analysis for a two dimensional plate; the boundary layer at the edge is set equal to a critical depth predicted by analogy to open channel hydraulics. In the center of the plate Singh's [6] and Clifton's prediction for the local heat transfer coefficient agree, but near the edge, significant discrepancies exist.

Rotem [8] questions the use of boundary layer approximations and uses photography to show that the thermal boundary increases with increasing Rayleigh number, contrary to the results of [5-7].

Clearly accurate experimental results are needed to

resolve some of the questions raised in the analytical works. Experimental heat transfer results are quite difficult to obtain for this problem because of the very modest value of the heat transfer coefficient from the horizontal surface. Heat losses through side walls and the plate supporting structure can be of the same order of magnitude as the heat transferred to the test plate. In addition, when the plate is used in air radiation heat transfer is significant: at low plate temperatures radiation absorbed from the surroundings is important, when the plate temperature is elevated, radiation emitted by the plate becomes important relative to convection. Therefore, it is difficult to accurately determine the net convective heat flux from the test plate by a heat balance.

Saunders, Fishenden and Mansion [9] measured the heat flux for a thin horizontal rectangular plate heated simultaneously on the top and the bottom in air. The average temperature gradient along the bottom surface was measured optically. It is likely that flow induced by the upper heated surface effected the results near the edge of the lower surface. Fuji and Imura [10] used parallel vertical side walls extending down from two sides of a heated horizontal rectangular plate in water in an attempt to create two dimensional flow. Since the distance between the side walls is one half the plate length the side walls effected the measurements; when the rectangular heated plate was turned to the vertical position the measured Nusselt number was seven per cent lower than theoretical value. Clifton, Allen and Erickson [11] used an energy balance to find the average Nusselt number for a horizontal heated plate. Details of the data reduction is not given but the scatter in the results points up the problems inherent in the heat balance method. It is not certain if the heated side walls of the horizontal plate were exposed or insulated for the tests in air. The results tend to fall ten percent below McAdams suggested correlation [12] based on Saunders data.

Abdulkadir [13] suspended a square horizontal plate in water. The vertical walls extending up from the edges of the plate were cooled by a water jacket to keep the wall temperature close to ambient. The water jacket overhung the edges of the plate somewhat. The average Nusselt number was determined by an energy balance. The results show considerable scatter probably due to the inaccuracies in estimating the heat flux to the cooling jacket. The local Nusselt number was found by measuring the local temperature distribution in the boundary layer and fitting the result to the form of temperature profile assumed by Singh [5]. The back of the test plate was grooved to reduce the longitudinal heat conduction and caused the

temperature to vary considerably over the surface of the plate. The boundary thickness was observed by using neutral density particles in water and by observing fog formation over a cooled horizontal plate facing upward.

Recently, Aihara, Yamada and Endo [14] measured the velocity distribution and temperature distribution under a heated rectangular plate. Their result closely agreed with the predictions of [6] near the center of the plate. Close to the edge the measured heat transfer coefficient is much higher than predicted [6]. The plate was heated on both sides and on the edge, this will cause a higher heat flux near the edge.

Aihara *et al.* carried their velocity measurements outside of the thermal boundary layer where they measured an appreciable horizontal flow from the edges toward the center.

To date the questions concerning heat transfer from a horizontal plate have not been accurately answered by the experimental results. This study is an attempt to provide some of the needed experimental information. In particular, the effect of edge conditions on the heat transfer from the plate surface is systematically studied. A heated square plate facing downward in air was used with three edge conditions: vertical walls extending up from the edge cooled to ambient temperature, vertical walls extending up from the edge heated to the plate temperature and adiabatic horizontal extensions added to the outer edge of the plate. The latter geometry although a rather extreme end condition, is the configuration used for a glass fiber spinning apparatus.

2. EXPERIMENTAL

Apparatus

A horizontal heated plate was placed face down in air contained in a large box ($4 \times 4 \times 5$ ft) with an open top. The sides of the box were closed to prevent extraneous convection. A square copper plate $7 \times 7 \times 1$ in. was used for the heated surface. The plate was thick to assure temperature uniformity over the test surface. Also threaded holes for the support bolts did not extend down to the lower surface and thereby distort the temperature uniformity. Seventeen thermocouples were placed in the plate at the lower surface. The plate was electrically heated by strip heaters; the heaters near the edge could be controlled independently of those in the center. Insulation and guard heaters were placed above the strip heaters. Thermocouples were placed on the support bolts near the plate heaters and near the guard heaters.

The three edge geometries used are shown in Fig. 1. In the first configuration, Fig. 1a, water cooled copper coils were placed around the four sides of the copper

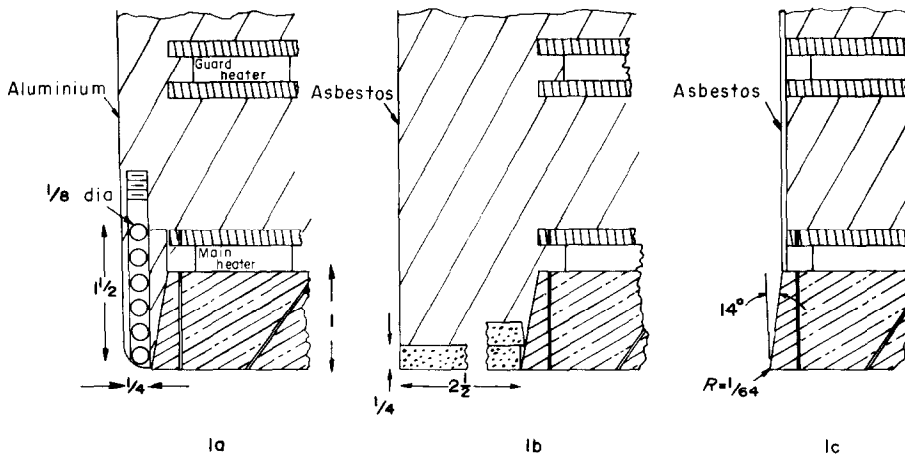


FIG. 1. The three edge configurations used in the present investigation, (a) Cooled sides, (b) Adiabatic extension, (c) Heated sides.

plate. The outside surface of the coils was covered with an aluminium plate. Thermocouples placed on the aluminium plate indicated that the sides always stayed within four degrees of ambient. A small overhang along the plate edge was needed to insure that the aluminium plate at the bottom was in contact with a cooling coil. The space between the coils and the test plate was filled with fibreglas insulation.

Figure 1b shows the second configuration. The coiling jacket was removed and asbestos extensions were added so that the outer edge of the extensions formed a square twelve inches on a side. The asbestos plates were carefully leveled with respect to the lower plane of the test plate. A very small gap, approximately 0.030 in., was left between the asbestos and the copper plate at the lower plane to minimize heat transfer to the edge of the asbestos. A second layer of asbestos (see Fig. 1b) touched the copper surface. Vertical side walls were placed at the outer edge of the extensions and the space inside the side walls was filled with fibreglas insulation.

In the third configuration, the sides of the plate were bare and vertical asbestos side walls were placed above the plate, Fig. 1c.

A very fine chromel-constantan thermocouple with a seven mill bead diameter was used to measure the temperature distribution in the boundary layer below the plate surface. The probe is shown schematically in Fig. 2. An electrical circuit was used to determine when the bead made contact with the plate surface. Optical measurements were not used because of possible scintillation effects. In addition to the probe, several thermocouples were kept inside the enclosure to monitor the ambient temperature.

Test procedure

The ambient temperature within the box and the plate temperature was monitored, when these temperatures did not change with time at a constant power input conditions were considered to be in steady state. After steady state conditions were achieved the temperature distribution across the plate surface was determined. The maximum temperature difference between the thermocouples in the plate was one half degree Fahrenheit. This difference is insignificant in comparison to the plate to ambient temperature difference which ranged from 50°F to 230°F . The plate was then checked to be sure the bottom surface was still horizontal with a precision water bubble level. The current to the guard heaters was adjusted so that the temperature difference between the guard heaters and plate was as small as possible.

Starting at the plate surface, the probe was traversed vertically downward and temperature measurements were made at each 0.005 in. interval. At least five intervals were used to determine the temperature gradient of the air at the plate surface. The thermal boundary layer thickness was determined by transverse the probe downward until the difference measured temperature and ambient temperature was approximately 0.2°F . It was difficult to accurately determine this location due to fluctuations in the temperature near the outer edge of the boundary layer. The boundary layer thickness and the air temperature gradient at the plate surface were measured at eight locations across the plate surface lying on a straight line from the center to the middle of one of the sides. Measurements were also made at six locations lying on a diagonal in three different tests.

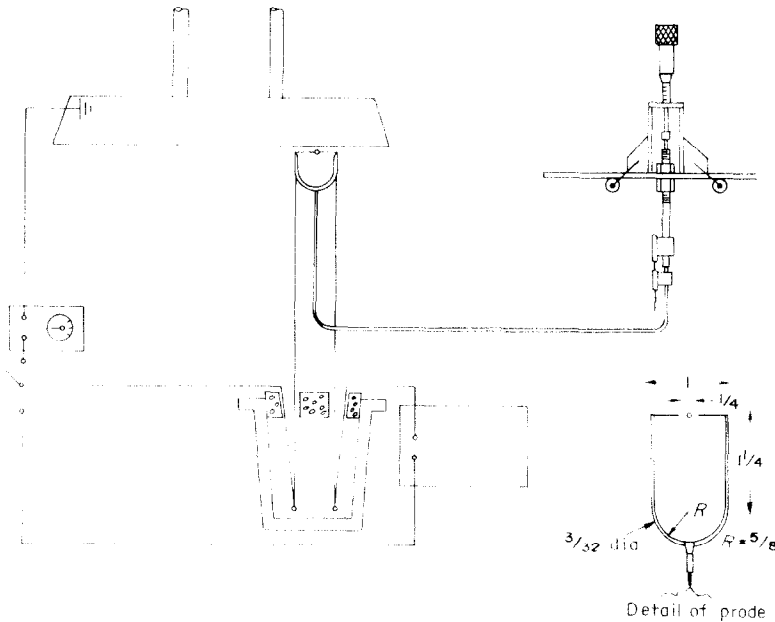


FIG. 2. Schematic of the temperature probe.

One measurement was made on the other half of the plate surface to confirm that symmetry existed. During all tests the ambient temperature varied by less than two degrees Fahrenheit. For the configuration with water cooled sides, shown in Fig. 1a, the cooled water flow rate and temperature rise were measured.

Calculations

The data taken with the probe was used to calculate the temperature gradient in the air at the plate surface. Since all of the measurements were kept close to the plate surface, at distances less than 4 per cent of the boundary layer thickness, the measured temperature distribution was linear. Radiation corrections to the probe readings were negligibly small. The local heat transfer coefficient was calculated for eight locations across the plate surface. In three tests, measurements at six diagonal locations and six lateral locations lying on a line from the center to the middle of one side were made. The uncertainty in the measured heat transfer coefficient was less than 2 per cent.

The heat transfer coefficient averaged over the entire surface was calculated from the local heat transfer coefficients assuming that constant coefficients form a square around the center of the plate. With this assumption the average coefficient based on the lateral measurements was within 1 per cent of the average coefficient based on the average of lateral

and diagonal measurements.

The average heat transfer coefficient can also be found from an energy balance. The emissivity of the plate was chosen so that the average heat transfer coefficients found by summing the local coefficient over the plate area agreed as closely as possible with the results from the heat balance for all of the tests with one edge configuration. The emissivity was assumed to obey the Hagen-Rubens emissivity relation but otherwise to remain constant from test to test. For some of the tests the average coefficient calculated from an energy balance disagreed with the average coefficient calculated from the local coefficients by as much as 15–20 per cent. The large scatter in the energy balance results from uncertainties in the calculation of heat transfer to the cooling coil, heat transfer between the plate and guard heater, and miscellaneous heat losses. Therefore the results of the energy balance were discarded and all of the average heat transfer coefficients presented in this paper are averaged local values. Although the local heat transfer coefficients are tedious to obtain, they yield accurate values for the average heat transfer coefficient.

In reducing the results to dimensionless form, the properties in the boundary layer were evaluated at the reference temperature recommended by Eckert and Drake [15]

$$T_w = 0.38(T_w - T_\infty).$$

3. RESULTS

The experimental results of the present work for water cooled vertical sides is compared with Abdulkadir's results and the theories of Clifton and Singh on Fig. 3. In the center of the plate, the present results are in remarkably good agreement with Singh's results for a square plate [5]. The agreement breaks down

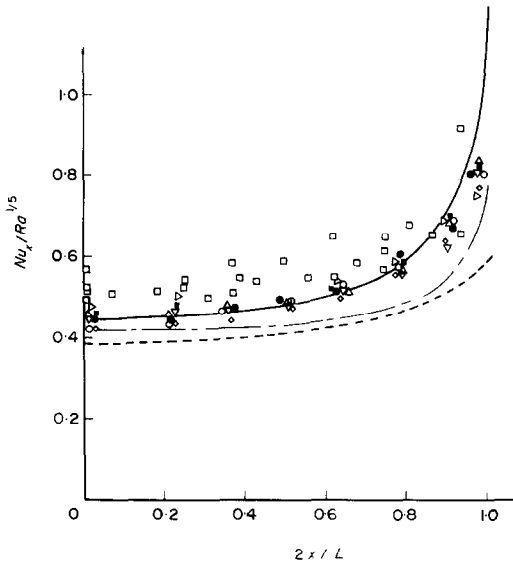
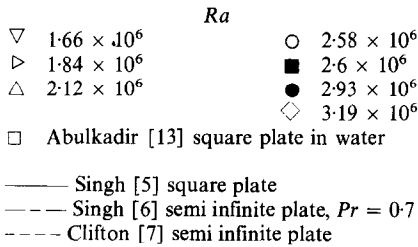


FIG. 3. Local Nusselt number distribution with cooled sides.



near the edge because of the erroneous theoretical assumption of zero boundary layer thickness at the outside. The disagreement at the outer portion of the plate surface where the plate area is largest results in a significant error in the predicted value of the average heat transfer coefficient. The theoretical results for a semi-infinite strip with non-zero boundary layer thickness at the plate edge underpredicts the heat transfer coefficient. This is in agreement with [5] which showed that a semi-infinite strip should have a lower heat transfer coefficient than a square plate at the same Rayleigh number. To date, analytical results for a square plate with a finite boundary layer thickness at the edge have not been obtained.

The thermal boundary layer thickness across the plate is compared to the available theories on Fig. 4.

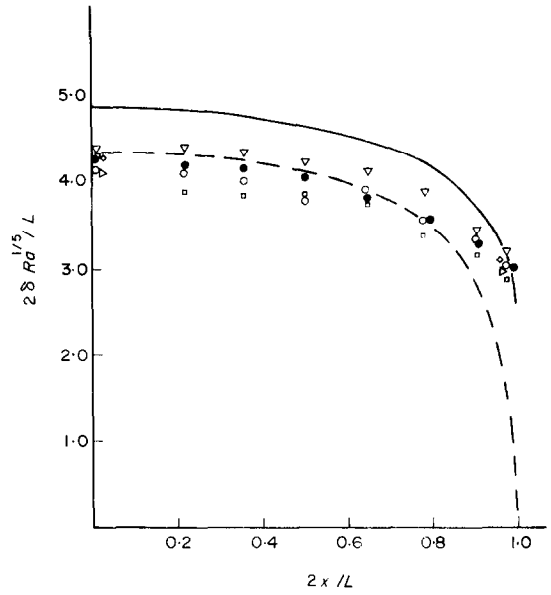


FIG. 4. Thermal boundary layer thickness with cooled sides.

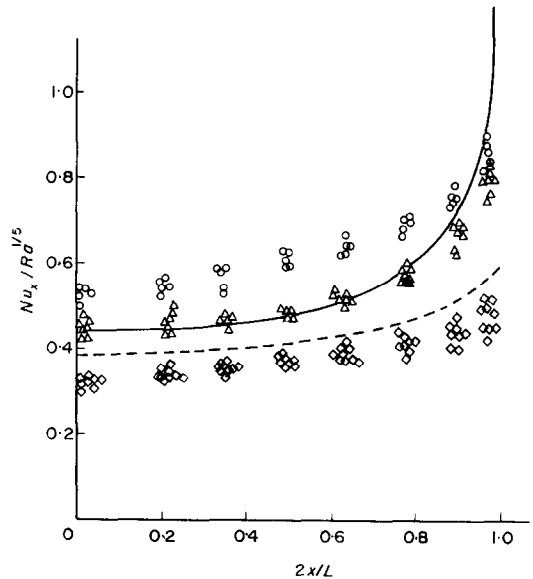
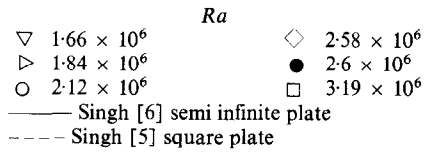
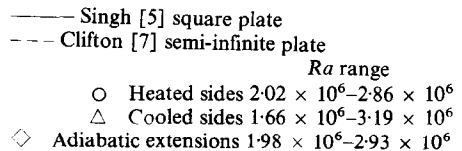


FIG. 5. Local Nusselt distribution for three edge conditions.



Again there is a large disagreement between the measurements and the theory near the plate edge. The scatter in the data reflects in part the uncertainty in measuring the boundary layer thickness; it also suggests that the form of the non-dimensional boundary layer thickness is not entirely correct.

The local heat transfer coefficient for heated sides and for adiabatic extensions are compared with the values for water cooled sides in Fig. 5. Changing the edge conditions appreciably changes the heat transfer over the entire surface of the plate. The heated sides augments the vertical plume rising from the side of the plate. The plume tends to draw the horizontal layer around the corner of the plate faster. Figure 6 bears this out, the boundary layer at the edge of the heated plate has been thinned and therefore accelerated by the action of the plume. The influence of the

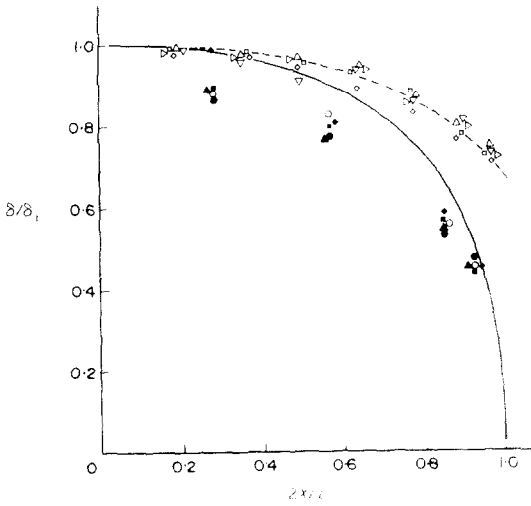


FIG. 6. Thermal boundary layer thickness for cooled sides and heated sides.

--- Clifton [7] Semi infinite plate	
- - - Singh [5] square plate	
Cooled sides	
▽	$Ra = 1.66 \times 10^6$
△	2.12×10^6
□	2.58×10^6
◇	2.93×10^6
▷	3.19×10^6
Heated sides	
○	$Ra = 2.02 \times 10^6$
●	2.44×10^6
◐	2.70×10^6
◑	2.86×10^6
▲	2.81×10^6

heated sides is accentuated in this experiment by the large area of the sides, equal to 59 per cent of the horizontal area.

The boundary thickness at the center of the plate is given in Table 1. Although it is impossible to define a hydrodynamic boundary layer at the plate center, a well-defined thermal boundary layer exists. At high Rayleigh numbers the heated sides cause a decrease

Table 1. Boundary layer thickness at the center of the plate

Cooled sides		Heated sides		Insulated extensions	
δ (in.)	Ra	δ (in.)	Ra	δ (in.)	Ra
0.83	1.66×10^6	0.81	2.02×10^6	0.94	2.42×10^6
0.82	2.12	0.75	2.44	0.81	2.92
0.80	2.58	0.68	2.70		
0.76	2.93	0.58	2.86		
0.68	3.19	0.55	2.81		

in the boundary layer thickness over the entire plate surface. The results shown in Table 1 reaffirm that the boundary layer thickness decreases with increasing Rayleigh number. The thickness of the boundary layer, almost one quarter of the plate half width, does bring into question some of the usual boundary layer assumptions. However, the agreement between the test results and the theory based on the boundary layer assumptions tends to answer this question.

It is doubtful if the geometry at the outermost edge of the plate has a significant influence on the flow since the boundary layer thickness is much greater than the radius of curvature.

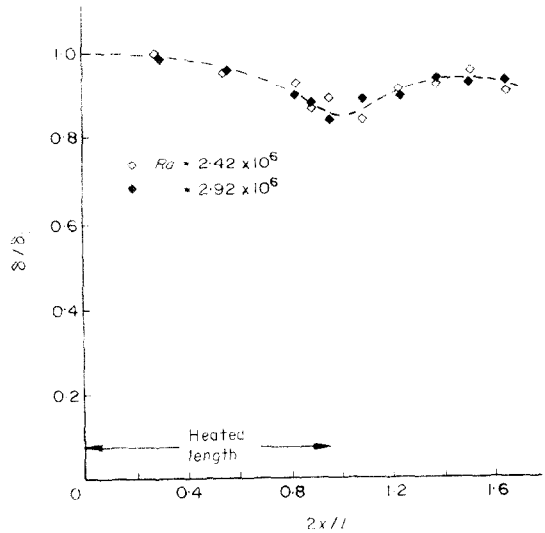


FIG. 7. Thermal boundary layer thickness with adiabatic extensions.

The addition of the horizontal extensions on the sides of the plate reduce the heat transfer coefficient by 25 per cent or more. This can be largely attributed to the increased flow resistance. The unexpected behavior of the boundary layer for this case is shown in Fig. 7. The boundary layer decreases in thickness from the center to the edge of the heated plate. Over the insulated section the boundary layer tends to grow again. The growth may be due to an interaction

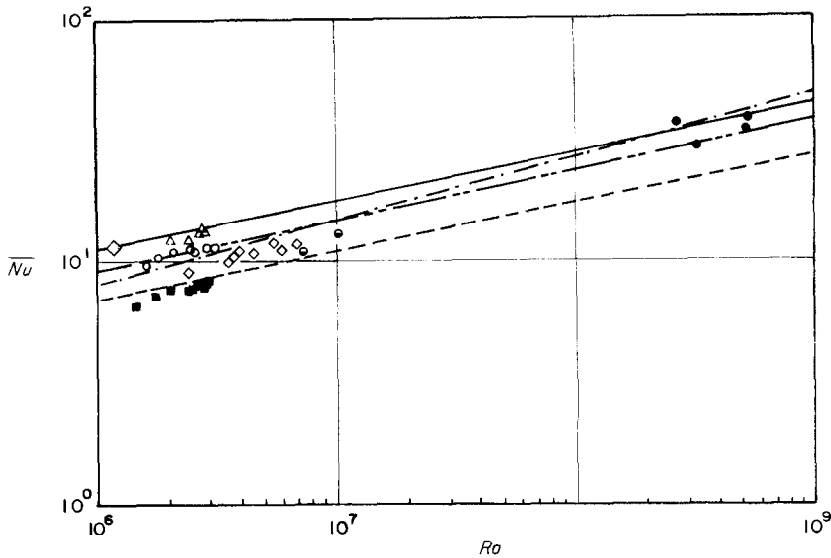


FIG. 8. Average Nusselt number vs Rayleigh number for square plates and rectangular plates. — Singh [5], - - - - McAdams [12], ····· Clifton, - · - · - proposed correlation. \diamond Clifton [7] rectangular plate in air, \bullet Abdulkadir [13] square plate in water, \bullet Aihara [14] rectangular plate in air, \blacksquare present data, use of extensions, \blacktriangle present data, heated sides, \circ present data, sides at ambient temp.

with the reverse flow originally outside the boundary layer or a redistribution of the velocity and temperature profiles.

The average Nusselt number for the heated plate is shown on Fig. 8 with other available data and some of the correlations proposed in the literature. The present data for a horizontal plate facing downward with cooled sides is correlated by

$$Nu = 0.587 Ra^{\frac{1}{4}} \quad (1)$$

It is unfortunate that more accurate data for water is not available to evaluate the proposed correlation at high Rayleigh numbers. The data for heated sides is correlated by

$$Nu = 0.68 Ra^{\frac{1}{4}} \quad (2)$$

while the data for the horizontal extensions is correlated by

$$Nu = 0.41 Ra^{\frac{1}{4}} \quad (3)$$

With the limited range of Rayleigh numbers explored in this investigation and the absence of accurate data at higher Rayleigh numbers in the literature, the exponents used in equations (1)–(3) are open to question especially if the correlations are extrapolated to higher Rayleigh numbers. The values of the exponents were chosen to agree with the theoretical values.

CONCLUSIONS

The conditions at the outer edge of a heated

horizontal plate facing downward have a first order influence on the thermal boundary layer shape and heat flux distribution over the entire heated area. Previous experimental investigations have used a variety of edge conditions making it difficult to obtain a single accepted correlation for the results. The available theoretical treatments do not consider the conditions at the edge in detail and the predicted behavior in the outer portions of the plate is approximate at best.

The boundary layer thickness approaches one quarter of the plate half width for this experiment. The usual boundary layer assumptions are open to question for these conditions. However, the generally good agreement between the data and theories based on the boundary layer assumptions near the center of the plate and the disagreement near the edges indicates that the edge conditions are the most important source of errors in the available analysis.

The difficulty in correlating available experimental results is compounded by the large uncertainty limits inherent with many of the techniques used to measure the heat flux. Energy balance methods are particularly error prone; measurements of local temperature gradients must be made to accurately determine the total heat flux from the plate surface.

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ÉFFET DES CONDITIONS DE BORD SUR LA CONVECTION NATURELLE SUR UNE PLAQUE HORIZONTALE

Résumé — On étudie la convection naturelle sur une plaque chaude horizontale tournée vers le haut dans l'air, avec trois conditions de bord différentes: surfaces verticales à la température ambiante surfaces verticales à la température de la plaque, surfaces horizontales adiabatiques dans le plan de la plaque. On détermine le flux de chaleur local en mesurant le gradient de température à la surface de la plaque. Le coefficient moyen de convection est obtenu à partir des valeurs locales.

Le coefficient moyen de convection pour le bord chauffé est supérieur de dix sept pour cent à celui qui correspond au bord froid. Pour l'extension horizontale de surface, ce coefficient est inférieur de trente pour cent à celui relatif au bord froid.

DER EINFLUSS DER RANDVERHÄLTNISSE AUF DIE NATÜRLICHE KONVEKTION EINER HORIZONTALEN PLATTE

Zusammenfassung — Es wurde die natürliche Konvektion an einer horizontalen, an der Unterseite beheizten quadratischen Platte, in Luft unter drei verschiedenen Randbedingungen untersucht. Kühlung der senkrechten Mantelfläche am Plattenrand auf Umgebungstemperatur, Aufheizung der senkrechten Mantelfläche am Plattenrand auf die Temperatur der Platte und adiabater horizontaler Verlauf in der Nähe des äusseren Plattenrandes. Der örtliche Wärmefluss wurde durch die Messung des Temperaturgradienten auf der Plattenoberfläche bestimmt. Der Wirtlere Wärmetransportkoeffizient wurde aus den örtlichen Werten ermittelt.

Der mittlere Wärmeübergangskoeffizient für den beheizten Rand war um 17% grösser als der für gekühlten. Der Wärmeübergangskoeffizient für den adiabaten Verlauf lag 30% unter dem des gekühlten Randes.

ВЛИЯНИЕ УСЛОВИЙ НА КРОМКЕ ПЛАСТИНЫ НА ЕСТЕСТВЕННУЮ КОНВЕКЦИЮ ОТ ГОРИЗОНТАЛЬНОЙ ПЛАСТИНЫ

Аннотация — Исследовалась естественная конвекция от обращенной вниз нагретой горизонтальной квадратной пластины в воздухе при трех различных условиях на кромке: поверхности вертикального выступа на кромке пластины охлаждались до температуры окружающей среды; температура поверхности вертикального выступа на кромке пластины поддерживалась равной температуре пластины; на горизонтальных выступах кромки пластины создавались адиабатические условия. Значения локального теплового потока находились по данным измерений температурного градиента на поверхности пластины. По локальным значениям получено среднее значение коэффициента переноса тепла.

Среднее значение коэффициента теплообмена для нагретой кромки было на 17% больше, чем для охлаждаемой кромки. Найдено, что коэффициент теплообмена для горизонтальных выступов на 30% ниже значения для охлаждаемой кромки.